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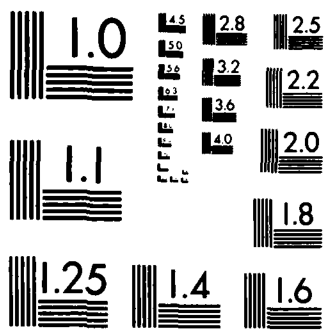

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Near resonance, the rate of change of the orbital inclination of an earth satellite may be expressed as an aggregate of several more or less complicated functions involving the parameters characterizing the resonance type, the orbital elements, certain quantities named "lumped coefficients," and time. The lumped coefficients are linear expressions consisting of the scalar products of sets of expansion coefficients of the earth's gravitational potential (in the sense that each lumped coefficient is associated

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with a specific potential term order) times a second set of parameters customarily called "influence numbers." The latter are essentially functions of orbital semimajor axis, inclination, and eccentricity. The present report specifies a computer algorithm for the influence numbers. Included are numerical trial data and a mathematical derivation of the influence number algorithm.

FOREWORD

This technical report documents a computer algorithm for the "influence numbers" that govern the relationship between certain subsets of the earth's gravitational potential expansion coefficients and "lumped coefficients," each of which summarizes the gravitational effect of an entire group of potential expansion coefficients on the inclination resonance of earth satellites.

The work was performed in the Space and Surface Systems Division as part of a larger effort, with the purpose of establishing equations of condition for groups of gravitational field parameters as a contribution to the validation of the WGS-84 Geodetic Parameter Solution.

The present report was reviewed by C. W. Duke, Jr., Head of the Space and Surface Systems Division; R. L. Kulp, Head of the Space and Ocean Geodesy Branch; and Dr. C. Oesterwinter, Research Associate.

Released by:

THOMAS A. CLARE, Head
Strategic Systems Department

CONTENTS

INTRODUCTION

An artificial earth satellite is said to be in β/α resonance* with the earth's gravitational field when it performs β nodal periods while the earth revolves α times relative to the satellite's precessing orbit plane. Near resonance, the perturbations caused by those tesseral components of the earth's gravitational potential with which the satellite happens to be in resonance generally exceed in magnitude all the remaining tesseral perturbations.

Near resonance, the rate of change of the orbital inclination of the satellite may be expressed as an aggregate of several more or less complicated functions involving the parameters characterizing the resonance type, the orbital elements, certain quantities named "lumped coefficients," and time. The lumped coefficients are linear expressions consisting of the scalar products of sets of expansion coefficients of the earth's gravitational potential (in the sense that each lumped coefficient is associated with a specific potential term order) times a second set of parameters customarily called "influence numbers." The latter are essentially functions of orbital semimajor axis, inclination, and eccentricity.

Satellites with sufficiently short orbital periods will, as their orbits decay, pass through resonance or, frequently, through a succession of different resonances. While orbital resonance effects are not restricted to the orbit inclination, conditions for acquiring useful inclination resonance data have been especially favorable, giving rise to a large body of investigations (References 1 through 78). Most of these are undertaking to evaluate the lumped coefficients for empirical time sequences of resonant inclination values. There is almost always an attempt made to subsequently infer numerical values for the geopotential expansion coefficients related to the lumped coefficients found. Most of the investigations seen are confined to specific resonance cases and to specific satellites. Some however attempt a synopsis (see especially Reference 6[illegible]). An effort was recently made at the Naval Surface Weapons Center (NSWC) to systematically review the entire body of published research work and to compile all lumped coefficients listed. A system of linear conditional equations complete with the customary statistical information was generated for the geopotential expansion

*For details, see the discussions of resonance in References [illegible] and [illegible].

coefficients, the latter being the system unknowns, and ranging up to degree 42, each identified lumped coefficient contributing one equation. A general, numerical solution for those geopotential parameters contributing to the inclination resonance cases treated in the just-mentioned collection of literature will in due course be executed.

As already stated, a necessary ingredient of the conditional equations are the influence numbers. These are, essentially, geometry factors reflecting the orbital parameters at resonance. As a generally valid theory of the influence numbers could not be located in any of our sources, it was derived starting from the Lagrange planetary equation for the inclination variation. To document the resulting computer algorithm is the purpose of the present report.

ALGORITHM FOR THE INFLUENCE NUMBERS

INPUT DATA

R ..... "Radius of the Earth" (semimajor axis of reference ellipsoid) in km. Default value: $R = 6378.140$ km.

a ..... Semimajor axis of satellite, in km.

e ..... Orbital eccentricity

i ..... Orbital inclination

β, α ..... Integers, characterizing β/α orbital resonance. Normally, $\alpha = 1$ or 2. In most cases, β will be larger than 5 and smaller than 50.

γ ..... Integer, denoting magnitude of the resonance term (order of "overtone"). Generally, γ will be equal to 1 or 2. Generally, it will be a positive integer.

q ..... Integer (positive, negative or zero) associated with the magnitude of the orbital eccentricity. For the present purpose, the following values of q may occur: 0, ± 1, ± 2, ± 3.

THE INFLUENCE NUMBERS

$$k = \alpha\gamma - q \tag{1}$$

$$m = \beta\gamma \tag{2}$$

$$K = (\beta - \alpha)\gamma + q \tag{3}$$

If K is even

$$\ell_r = \beta\gamma + 2r = m + 2r \tag{4}$$

$$m_r = \beta\gamma \tag{5}$$

$$p_r = \frac{1}{2}(\beta - \alpha)\gamma + \frac{1}{2} q + r \tag{6}$$

$$= \frac{1}{2} K + r \tag{6a}$$

If K is odd

$$\ell_r = \beta\gamma + (2r + 1) = m + 1 + 2r \tag{7}$$

$$m_r = \beta\gamma \tag{8}$$

$$p_r = \frac{1}{2}(\beta - \alpha)\gamma + \frac{1}{2} q + \frac{1}{2} + r \tag{9}$$

$$= \frac{1}{2}(K + 1) + r \tag{9a}$$

$$r = 0, 1, 2, 3, 4, \ldots\ldots\ldots\ldots, r_{max} \tag{10}$$

Expected maximum value for r_{max}

$$r_{max} \leq 50 \tag{11}$$

The influence numbers are then

$$Q_{\ell}^{q,k} = (-1)^{r}\left(\frac{R}{a}\right)^{2r} \frac{G_{\ell pq}}{G_{\ell_0 p_0 q}} \; \frac{FAN_{\ell mp}}{FAN_{\ell_0 m p_0}} \tag{12}$$

where

$$\ell = \ell_r \tag{13a}$$

$$p = p_r \tag{13b}$$

$$\ell_0 = \ell(r = 0) \tag{13c}$$

$$p_0 = p(r = 0) \tag{13d}$$

Note that the influence numbers are dimensionless quantities.

INCLINATION FUNCTION SUBROUTINE

$$FA_{\ell mp} \equiv FA_{\ell mp}(i) \equiv \frac{(\ell + m)!}{2^{\ell} p!(\ell - p)!} * \sum_{\eta_{min}}^{max} \left[(-1)^{\eta} \binom{2\ell-2p}{\eta} \binom{2p}{\ell-m-\eta} * C^{3\ell-m-2p-2\eta} * S^{m-\ell+2p+2\eta} \right] \tag{14}$$

$$C = \cos\left(\frac{1}{2}\, i\right) \tag{15}$$

$$S = \sin\left(\frac{1}{2}\, i\right) \tag{16}$$

$$\eta_{min} = \max[0, (\ell-m-2p)] \tag{17}$$

$$\eta_{max} = \min[(\ell-m), (2\ell-2p)] \tag{18}$$

$$N_{\ell m} = + \sqrt{\frac{(\ell - m)!(2\ell + 1)(2 - \delta_m^0)}{(\ell + m)!}} \tag{19}$$

$$\delta_m^0 = 0 \quad \text{for} \quad m \neq 0 \tag{20a}$$

$$= 1 \quad \text{for} \quad m = 0 \tag{20b}$$

$$FAN_{\ell mp} = N_{\ell m} * FA_{\ell mp} \tag{21}$$

ECCENTRICITY FUNCTION SUBROUTINE

$$G_{\ell pq} \equiv G_{\ell pq}(e) \equiv (-1)^{|q|}(1 + \beta^2)^{\ell}\, \beta^{|q|}$$

$$* \sum_{\kappa=0}^{N} V_{\ell pqk} W_{\ell pqk} \beta^{2k} \tag{22}$$

$$\beta = \frac{e}{1 + \sqrt{1 - e^2}} \tag{23}$$

Stop summation over k when $[G]_{N+1}$ and $[G]_N$ agree to six significant figures.

$$\text{For} \quad p \leq \ell/2, \quad p' = p, \quad q' = q \tag{24}$$

$$\text{For} \quad p > \ell/2, \quad p' = \ell - p, \quad q' = -q \tag{25}$$

$$V_{\ell pqk} \equiv \sum_{r=0}^{h} \binom{2p' - 2\ell}{h - r} \frac{(-1)^r}{r!} \left(\frac{(\ell - 2p' + q')e}{2\beta} \right)^r \tag{26}$$

For $q' > 0, \quad h = k + q'$ (27)

For $q' \leq 0, \quad h = k$ (28)

$$W_{\ell pqk} \equiv \sum_{r=0}^{h} \binom{-2p'}{h - r} \frac{1}{r!} \left(\frac{(\ell - 2p' + q')e}{2\beta} \right)^r \tag{29}$$

For $q' \geq 0, \quad h = k$ (30)

For $q' < 0, \quad h = k - q'$ (31)

To avoid difficulties arising from use of negative arguments, calculate the binomial coefficients occurring in Equations 26 and 29 as follows:

$$\binom{A}{B} = \frac{1}{B!} \left\{ [A - 0]*[A - 1]*[A - 2]* \;\ldots\ldots\; *[A - (B - 1)] \right\} \tag{32}$$

B may be expected to be a positive integer always. A may assume negative values.

ALGORITHM FOR THE LUMPED COEFFICIENTS

Let $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$ be the normalized expansion coefficients of the geopotential and $C_{\ell m}$ and $S_{\ell m}$ their unnormalized counterparts, then

$$\bar{C}_{\ell m} = C_{\ell m}/N_{\ell m} \tag{33}$$

$$\bar{S}_{\ell m} = S_{\ell m}/N_{\ell m} \tag{34}$$

APPENDIX B

MATHEMATICAL BACKGROUND

R (km)	6378.135	6378.135
i (deg)	38.92	74.02
a (km)	7403.314	8095
e	.082	.06
ℓ	13	29
[illegible]	1	2
[illegible]	2	1
q	0	0
r	1	1
K	24	27
m	26	29
C_0	26	30
P_0	12	14
C	28	32
P	13	15
k	2	2
FAN	0.000413812	-0.021333873
FAN_q	0.000022206133	0.434536005
G	2.966430720	2.218928773
G_0	2.613511626	2.039722371
$Q(\ell,q,k)$	-15.6991063	0.03315660

R (km)	6378.135	6378.135	6378.135
i (deg)	32.97	51.6	51.6
a (km)	6902.251	9714.537	9714.537
e	.003	.180	.180
[illegible]	15	9	9
[illegible]	1	1	1
[illegible]	1	1	1
q	0	-1	-3
r	6	3	3
K	14	7	5
m	15	9	9
ℓ_0	15	10	10
P_0	7	4	3
ℓ	27	16	16
P	13	7	6
k	1	2	4
FAN	0.396980759	-0.312034989	0.[illegible]42922387
FAN_0	0.000215955[illegible]	0.501373731	0.643760559
[illegible]	1.00169282[illegible]	3.6100[illegible]8758	0.[illegible]80926
[illegible]	1.000531091	1.050359[illegible]	[illegible]39859[illegible]
[illegible]	[illegible].41966	0.1712[illegible]417	[illegible]711307

APPENDIX A

NUMERICAL TRIAL DATA FOR THE INFLUENCE NUMBER ALGORITHM

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where $N_{\ell m}$ is the normalization factor specified by Equation 19. The relationship among the lumped coefficients, $\bar{\bar{C}}_m^{q,k}$ and $\bar{\bar{S}}_m^{q,k}$, the potential expansion coefficients and the influence numbers is

$$\bar{\bar{C}}_m^{q,k} = \sum_{r=0}^{rMAX} Q_\ell^{q,k} \; \bar{C}_{\ell m} \tag{35}$$

$$\bar{\bar{S}}_m^{q,k} = \sum_{r=0}^{rMAX} Q_\ell^{q,k} \; \bar{S}_{\ell m} \tag{36}$$

Note once more that the influence numbers as well as the potential expansion coefficients are dimensionless.

The following derivation should be viewed in context with the introduction to this report. Lagrange's planetary equation for the inclination variation was adopted from Pages 287 and 288 of Reference 33 and reads as follows:

$$\frac{di}{dt} = \frac{\dot{M}}{\sqrt{1 - e^2}} * \sum_{\ell mpq} \left\{ J_{\ell m}\left(\frac{R}{a}\right)^{\ell} * \mathrm{Re}\left[j[(\ell - 2p) \operatorname{cotg} i - m \operatorname{cosec} i] \right. \right.$$

$$\left. \left. * F_{\ell mp}(i)\ G_{\ell pq}(e)\ \exp(j\psi_{\ell mpq}) \right] \right\} \tag{B01}$$

$$\psi_{\ell mpq} = (\ell - 2p)\omega + (\ell - 2p + q)M + m(\Omega - S) - m\lambda_{\ell m} \tag{B02}$$

$$J_{\ell m} = \sqrt{C_{\ell m}^2 + S_{\ell m}^2} \tag{B03}$$

$$m\lambda_{\ell m} = \tan^{-1}(S_{\ell m}/C_{\ell m}) \tag{B04}$$

where a, e, i, ω, Ω are the familiar orbital elements, $\dot{M}$ is the mean motion, R is the equatorial "radius of the earth," M is the mean anomaly, S is the sidereal Greenwich time of the epoch of the elements, and $C_{\ell m}$ and $S_{\ell m}$ are the unnormalized expansion coefficients of the gravitational potential.

Also from Page 288 of Reference 33, for [illegible] resonance there hold the relationships

$$\psi_{\ell,[illegible],p,q} = [illegible] - q [illegible] - m\lambda_{\ell m}$$ [illegible]

[illegible] = [illegible]([illegible] + M) + [illegible]([illegible] - S) [illegible]

k = ℓ - 2p [illegible] - q [illegible]

m = [illegible] [illegible]

as well as Equations 3 through 11 from the main body of this report. For the meaning of beta, alpha, gamma, and q see the segment "Input Data" from the "Algorithm for the Influence Numbers." Also note that

$$0 \leq p \leq \ell \tag{B9}$$

$$\ell \geq m \tag{B10}$$

Again from Page 288 of Reference 33, the Inclination Function, $F_{\ell mp}(i)$, is

$$F_{\ell mp}(i) \equiv j^{(\ell-m)} \; FA(\ell,m,p,i) \tag{B11}$$

where the function FA is specified by Equations 14 through 16 above.

$$\overline{F}_{\ell mp}(i) \equiv j^{(\ell-m)} \; FAN(\ell,m,p,i) \tag{B12}$$

is the Normalized Inclination Function. The function FAN is the normalized version of FA and is defined by Equations 18 through 21.

$G_{\ell pq}(e)$ is the Eccentricity Function. It will be found defined on Page 37 of Reference 79 and is for our purposes specified by Equations 22 through 32 above. It shall already now be stated that the normalized expansion coefficients of the geopotential, $\overline{C}_{\ell m}$ and $\overline{S}_{\ell m}$, and the influence numbers, $C_m^{q,k}$ and $S_m^{q,k}$, are explained by Equations 33 through 36 in the main part of this report.

Now consider the portion

$$[illegible]_{\ell m} * Re[.....] = [illegible] * FA * [illegible]$$

$$* [illegible]_{\ell m} * Re[[illegible]] \tag{B13}$$

[illegible]

For $(\ell - m)$ equal to an even number:

$$J_{\ell m} * \mathrm{Re}[.....] = J_{\ell m} j^{(\ell-m)} \mathrm{Re}(j \exp(j\psi_{\ell mpq}))$$

$$= -J_{\ell m} j^{(\ell-m)} \sin \psi_{\ell mpq} = -J_{\ell m} j^{(\ell-m)} \sin (\gamma\Phi_\beta - q\omega) \cos m\lambda_{\ell m}$$

$$+ J_{\ell m} j^{(\ell-m)} \cos (\gamma\Phi_\beta - q\omega) \sin m\lambda_{\ell m} \qquad \text{(B14)}$$

$$m\lambda_{\ell m} = \tan^{-1}(S_{\ell m}/C_{\ell m}) \qquad \text{(BC4)}$$

$$\begin{pmatrix} \overline{C}_{\ell m} \\ \overline{S}_{\ell m} \end{pmatrix} = \frac{1}{N_{\ell m}} \begin{pmatrix} C_{\ell m} \\ S_{\ell m} \end{pmatrix} \qquad \text{(B15)}$$

$$\cos m\lambda_{\ell m} = \frac{1}{\sqrt{1 + S_{\ell m}/C_{\ell m}}} = N_{\ell m} \overline{C}_{\ell m}/J_{\ell m} \qquad \text{(B16)}$$

$$\sin m\lambda_{\ell m} = N_{\ell m} \overline{S}_{\ell m}/J_{\ell m} \qquad \text{(B17)}$$

and, finally,

$$J_{\ell m} * \mathrm{Re}[.....] = -N_{\ell m} j^{(\ell-m)} [\overline{C}_{\ell m} \sin (\gamma\Phi_\beta - q\omega) - \overline{S}_{\ell m} \cos (\gamma\Phi_\beta - q\omega)] \qquad \text{(B18)}$$

where

$$j^{(\ell-m)} = +1 \text{ or } -1 \qquad \text{(B19)}$$

For $(\ell - m)$ equal to an odd number, one obtains in a similar fashion

$$J_{\ell m} * \mathrm{Re}[.....] = N_{\ell m} j^{(\ell-m+1)} [\overline{C}_{\ell m} \cos (\text{[illegible]}) + \overline{S}_{\ell m} \text{[illegible]}] \qquad \text{(B20)}$$

where

$$j^{(\ell-m+1)} = +1 \text{ or } -1 \qquad \text{(B21)}$$

If one now enters Equations B18 and B20 respectively into Equation B13 and subsequently Equation B13 into Lagrange's planetary equation (B6i), observing that under the summations occurring in the latter equation m and q are constants while the summations over ℓ and p are according to Equations 4 and 9a subject to the common summation index r, there result expressions for the inclination variation where the geopotential expansion coefficients appear in groups that may be conveniently "lumped together" into the lumped coefficients $\bar{\bar{C}}_m^{q,k}$ and $\bar{\bar{S}}_m^{q,k}$ in a linear fashion.

For even K

$$\frac{di}{dt} = \frac{\dot{M}}{\sqrt{1 - e^2}} \left(\frac{R}{a}\right)^{\ell_0} A_{mk}(i)\, G_{\ell_0 p_0 q_0}(e)\, FAN_{\ell_0 m p_0}(i)$$

$$* \left[\bar{\bar{C}}_m^{qk} \sin(\gamma\Psi - q\omega) - \bar{\bar{S}}_m^{qk} \cos(\gamma\Psi - q\omega)\right] \qquad (B22)$$

For odd K

$$\frac{di}{dt} = \frac{\dot{M}}{\sqrt{1 - e^2}} \left(\frac{R}{a}\right)^{\ell_0} A_{mk}(i)\, G_{\ell_0 p_0 q_0}(e)\, FAN_{\ell_0 m p_0}(i)$$

$$* \left[\bar{\bar{C}}_m^{qk} \cos(\gamma\Psi - q\omega) \text{ [illegible]}\right] \qquad \text{[illegible]}$$

[illegible]

$$A_{mk}(i) = \text{[illegible]} \qquad \text{[illegible]}$$

$$\text{[illegible]} = \sum_{r=0}^{\max} \text{[illegible]} \qquad \text{[illegible]}$$

$$\text{[illegible]} = \sum_{r=0} \text{[illegible]} \qquad \text{[illegible]}$$

$$Q_\ell^{qk} = j^{2r}\left(\frac{R}{a}\right)^{2r} \frac{G_{\ell pq}}{G_{\ell_o p_0 q}} \; \frac{FAN_{\ell mp}}{FAN_{\ell_0 m p_0}} \qquad (B27)$$

$$j^{2r} \equiv (-1)^r \qquad (B28)$$

where the Q_ℓ^{qk} are the influence numbers for both even and odd values of k.

It should finally be stated that Equation B11 represents the Inclination Function as defined by Allan (Reference 1, p. 1830). Kaula's Inclination Function (Reference 79) "FK" may be calculated as follows:

For even values of $(\ell - m)$, including $(\ell - m) = 0$,

$$FK = j^{(\ell-m)}\ FA \qquad (B29)$$

For odd values of $(\ell - m)$

$$FK = j^{(\ell-m)}\ FA/(-j) = j^{(\ell-m+1)}\ FA \qquad (B30)$$

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